Evaluation of Endurance Time Analysis in Optimization of Viscous Damper Placement for Seismic Design

Hamed Amouzegar¹, Hossein Tajmir Riahi²*, and Maryam Daei²

1. Graduate Student, University of Isfahan, Iran
2. Assistant Professor, Civil Engineering Department, University of Isfahan, Iran,
* Corresponding Author; email: tajmir@eng.ui.ac.ir

Received: 14/08/2013
Accepted: 21/05/2014

ABSTRACT

Optimal placement of dampers with proper characteristics is the major concern for seismic rehabilitation of structures using these passive devices. In this paper, endurance time method is used to find the optimum placement of dampers in steel frames at different seismic hazard levels. Genetic algorithm is combined with ET method to overcome this optimization problem. Different types of nonlinear modeling are used to validate ET analysis results. ET acceleration functions used in this study are generated based on ASCE7-05 design spectrum. Results of ET analysis are compared with the results of time history analysis using seven near-field ground motions spectrally matched to the same spectrum. The Maximum difference between the results of two methods is about 15% in this study, and results of ET analysis are compatible with the results of time history analysis at different hazard levels. Consistency of the results shows that ET method can be used as an effective procedure to optimize damper placement in structures reducing the required computational effort.

1. Introduction

Energy absorption devices can be used effectively to reduce the inflicted damage to buildings after strong earthquakes, but choosing the best place for the dampers and assigning proper characteristics for them are the major concern for the optimum design, because these factors efficiently change the structural response during an earthquake [1]. However, seismic guidelines do not recommend a methodology to obtain optimal distribution of dampers [2].

Researchers have proposed different damper placement methods in recent decades. These methods include simple ones such as uniform and stiffness proportional damping distribution methods and more complicated ones such as sequential search algorithm [3, 4], gradient-based search [5], minimum transfer functions [6-8], fully-stressed analysis/redesign [9], genetic algorithms [10, 11] and active control based methods [12]. A good description about all of the proposed methods can be found in Whittle et al. paper [2]. Some of these methods are restricted to linear systems [13] and a quantity of them use simple assumptions to overcome solution problems [14]. Considering new seismic guidelines which use performance-based design criteria, new damper placement methods should be offered to assess the efficiency of damping distribution for satisfying different design criteria.

Damper placement can be examined properly if
all significant parameters are applied to structural models and nonlinear dynamic analysis with a set of ground motions is used. In reality, for complicated structures such as tall buildings or irregular structures, this evaluation is more time-consuming. This gets harder when the performance of the equipped structure should be verified at different excitation levels. Riahi et al. used Endurance Time (ET) method, a dynamic pushover analysis with reduced computational attempt, for optimal damper placement and showed its capability with a simple example [15]. Estekanchi and Basim tried to find the optimum arrangement of dampers in steel frames, which satisfy performance objectives using ET method and genetic algorithm as the optimization tool. Life Safety (LS) and Collapse Prevention (CP) were the two code performance objectives selected by them to be assured for two levels of excitation. They showed that ET analysis can correctly estimate the behavior of equipped frames at different seismic excitation [16].

In this paper, a more comprehensive study is done to assess the capabilities of ET method and genetic algorithm to find the optimum placement of dampers in steel frames. Performance of the frames is evaluated at different seismic hazard levels which facilitate application of ET method for performance-based seismic design. Moreover, different nonlinear models including both distributed and concentrated plasticity with and without stiffness and strength deterioration are assigned to the structures to see whether ET method can be used appropriately in more complicated situations. Ground motions used for evaluation of ET method are matched to the ASCE7-05 spectrum; and therefore, ET method can be verified directly and easily with the code hazard levels [17]. Results of two analyses are compatible for all frames and also ET method can properly predict the distribution of interstory drift ratios at different stories.

2. ET Acceleration Functions and Spectrum Matched Records

Endurance time method is basically a dynamic pushover method that tries to predict seismic response of structures by analyzing their performance when subjected to predesigned intensifying acceleration functions [18]. If ET acceleration functions have the major characteristics of seismic records with different intensities, they can efficiently estimate the real response of the structures to be used in performance-based engineering [19]. Therefore, this method can be used to check the performance of different systems at different excitation levels and find one with the optimum performance. In this paper, this capability is used to optimize the amount of damping devices and their placement. To be sure about the estimation of ET analysis, its results should be compared with the results of nonlinear time history analysis at different seismic levels. Therefore, Incremental Dynamic Analysis (IDA) is used in this research as a benchmark for comparison [20].

A major factor in the success of the endurance time method is in the availability of suitable intensifying excitation functions (i.e. acceleration functions in this study). Previous studies show that appropriate acceleration functions can actually be generated with the concept of response spectrum [16, 18, 21, 22]. In this way, acceleration functions are designed in such a way that each time window from zero to time $t$ of them produces a response spectrum which is proportional to the target spectrum multiplied by a scaling factor. This factor is linearly increasing with time and is equal to unity at a particular time called $t_{Target}$ [18]. To achieve this goal, the target acceleration response of ET acceleration function at different times can be defined as:

$$S_{at}(T,t) = \frac{I}{I_{Target}} S_{ac}(T)$$

In this equation, $S_{at}(T,t)$ is the target acceleration response spectrum at time $t$ and $S_{ac}(T)$ is the target acceleration response spectrum. Target response spectrum used for the generation of acceleration functions in this study (ETA40g series) is the design spectrum of ASCE7-05 for soil type C with $S_s = 1.5$, $S_l = 0.6$, $F_a = 1.0$, $F_v = 1.3$ and $T_L = 8$ [17]. Damping is considered to be 5% and $t_{Target}$ is 10 seconds. An optimization procedure is used to generate acceleration functions which their spectrum for any time window from zero to other time has a linear relation with the target spectrum as shown in equation 1. For example, for time 5 sec or 15 sec the spectrum will be equal to 0.5 or 1.5 times of target spectrum. Optimization of these acceleration functions is for 200 periods from 0 to 5 seconds plus 20
periods from 6 to 50 seconds. The total time of these acceleration functions is 40.96 seconds. This series consists of three acceleration functions that their initial values used at the beginning of the optimization procedure are different [21].

Seven near-fault ground motions (OGM set) were selected from the series of fourteen near-field records with pulses subset listed in FEMA P695 [23]. The specifications of ground motions of this set are shown in Table (1). The main goal of selecting near-field records in this study is to evaluate ET method for the worst condition. Ground motions with pulses can generate much higher Engineering Demand Parameters (EDP); and hence, the largest differences between ET analysis results and time history analysis results can be observed [24]. Of course, response spectra of these records are totally different from the target response spectrum; and therefore, they should be modified in order to make the comparison logical. As a result, records have been spectrum matched to the target response spectrum [25]. To keep the characteristics of ground motions properly, spectrum matching procedure is applied in time domain using wavelet method. This procedure has the advantage that the shape and frequency content of time series after spectrum-matching are almost unchanged [26]. Wavelets used in this study are reverse impulse and tapered cosine presented by Abrahamson [27]. After using reverse impulse function, for prevention of non-zero displacement and velocity at the end of the time series, a linearly base line correction has been applied to the spectrum matched results [25].

Figure (1) shows the arias intensity, Fourier amplitude and acceleration time history of record

Table 1. Description of the OGM set of ground motions used in this study.

<table>
<thead>
<tr>
<th>Date</th>
<th>Name</th>
<th>Magnitude (Ms)</th>
<th>Station Name</th>
<th>PGA (g)</th>
<th>Component (Degree)</th>
<th>Epicentral (km)</th>
<th>Abbreviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1999</td>
<td>Duess, Turkey</td>
<td>7.1</td>
<td>Duess</td>
<td>0.52</td>
<td>270</td>
<td>1.6</td>
<td>DZC</td>
</tr>
<tr>
<td>1992</td>
<td>Erzincan, Turkey</td>
<td>6.7</td>
<td>Erzincan</td>
<td>0.49</td>
<td>N-S</td>
<td>9</td>
<td>ERZ</td>
</tr>
<tr>
<td>1979</td>
<td>Imperial Valley-06</td>
<td>6.5</td>
<td>El Centro #6</td>
<td>0.44</td>
<td>230</td>
<td>27.5</td>
<td>H-E</td>
</tr>
<tr>
<td>1999</td>
<td>Kocaeli, Turkey</td>
<td>7.5</td>
<td>Izmit</td>
<td>0.22</td>
<td>90</td>
<td>5.3</td>
<td>ILZI</td>
</tr>
<tr>
<td>1992</td>
<td>Landers</td>
<td>7.3</td>
<td>Lucerne</td>
<td>0.79</td>
<td>0</td>
<td>44</td>
<td>LCN</td>
</tr>
<tr>
<td>1992</td>
<td>Cape Mendocino</td>
<td>7</td>
<td>Petrolia</td>
<td>0.63</td>
<td>90</td>
<td>4.5</td>
<td>PET</td>
</tr>
<tr>
<td>1994</td>
<td>Northridge-01</td>
<td>6.7</td>
<td>Rinaldi Receiving Sta</td>
<td>0.87</td>
<td>228</td>
<td>10.9</td>
<td>RRS</td>
</tr>
</tbody>
</table>
DZC before and after spectrum matching. Shape of the record after matching is similar to the original record, and increasing trend of arias intensity is comparable for both records. Fourier amplitude is changed for low frequencies, but the general tendency of the records is analogous. Figure (2) shows the mean acceleration response spectrum of ETA40g series at $t=10$ and 15 seconds. Besides, acceleration response spectrum of OGM set and acceleration response spectrum of matched set (GM) with scale factor 1 and 1.5 are depicted for comparison. As it can be seen, the compatibility of the GM set, ETA40g series and target response spectrum are acceptable. To compare the results of ET and time-history analysis at different seismic excitation levels, a relation between each time of ET analysis and each scale factor for time history analysis should be established. The scale factors for GM set and equivalent time of ET acceleration functions proportionate with the earthquake hazard levels and their corresponding performance levels (rehabilitation objective) are shown in Table (2).

3. Nonlinear Structural Models

In this study, eight steel moment resisting frames are considered, Table (3). The generic frames are...
Evaluation of Endurance Time Analysis in Optimization of Viscous Damper Placement for Seismic Design

Table 2. Scale factors for GM set and equivalent time of ET acceleration functions.

<table>
<thead>
<tr>
<th>Rehabilitation Objective</th>
<th>Scale Factor</th>
<th>ET Time (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Life Safety (LS) for BSE-1 (Basic Safety Earthquake 1)</td>
<td>1.00</td>
<td>10</td>
</tr>
<tr>
<td>Collapse Prevention (CP) for BSE-2 (Basic Safety Earthquake 2)</td>
<td>1.50</td>
<td>15</td>
</tr>
</tbody>
</table>

Table 3. Specification of the frames.

<table>
<thead>
<tr>
<th>Frames</th>
<th>Number of Stories</th>
<th>Number of Bays</th>
<th>Total Mass (kN(\cdot)m)</th>
<th>Period of Free Vibration (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SJB1</td>
<td>3</td>
<td>1</td>
<td>81.47</td>
<td>1.20</td>
</tr>
<tr>
<td>S3B1-SSD</td>
<td>3</td>
<td>1</td>
<td>81.47</td>
<td>1.17</td>
</tr>
<tr>
<td>S3B3</td>
<td>3</td>
<td>3</td>
<td>244.40</td>
<td>1.25</td>
</tr>
<tr>
<td>S3B3-SSD</td>
<td>3</td>
<td>3</td>
<td>244.40</td>
<td>1.16</td>
</tr>
<tr>
<td>S7B1</td>
<td>7</td>
<td>1</td>
<td>190.09</td>
<td>2.03</td>
</tr>
<tr>
<td>S7B1-SSD</td>
<td>7</td>
<td>1</td>
<td>190.09</td>
<td>1.97</td>
</tr>
<tr>
<td>S7B3</td>
<td>7</td>
<td>3</td>
<td>570.28</td>
<td>2.05</td>
</tr>
<tr>
<td>S7B3-SSD</td>
<td>7</td>
<td>3</td>
<td>570.28</td>
<td>1.87</td>
</tr>
</tbody>
</table>

The behavior of the steel material is considered as elastic with strain hardening. This material model has yielding stress of \( F_y = 240 \text{ MPa} \), elastic modulus of \( E = 206 \text{ GPa} \) and post-yield stiffness equal to 3% of the initial elastic stiffness. For four frames, these properties are applied in the analysis using OpenSees nonlinear beam-column elements with nonlinear distributed plasticity [31]. Other frames have concentrated plasticity using zero length elements at the joints. Stiffness degradation and strength deterioration of steel connections are applied to these elements using Ibarra and Krawinkler model [32]. These frames have SSD abbreviation in their names. These two types of modeling make it possible to evaluate the capabilities of ET analysis for assessment of different nonlinear models.

In recent years, engineers have applied systems that use viscous dampers to reduce lateral induced vibrations. Viscous dampers produce forces related to velocity. In this study, viscous dampers are modeled as bracing elements and viscous material available in OpenSees is used.

4. Optimization Procedure for Different Performance Objectives

The problem is to find optimized damper coefficients in each story so that the MIDR of the frame in two seismic hazard levels, Table (2), satisfy the performance objectives of ASCE41-06 (MIDR = 2.5% for design earthquake and MIDR = 2.5 or 5% for Maximum Considered Earthquake (MCE)).
It should be mentioned that any number of hazard levels and performance objectives can be applied in the optimization procedure and advantages of ET analysis can be seen in more complicated problems. At first step, some primary damper coefficients should be assigned to the dampers at different stories.

Next, frames are analyzed using OpenSees software and MIDR is found and verified with the desired values of ASCE41-06. The optimization problem is to find minimum damper coefficients at different stories so that performance objectives are satisfied at different hazard levels.

Heuristic optimization approaches have been widely used in seismic optimization problems in recent years [8]. In this paper, the genetic algorithm that is a popular heuristic method is utilized [10]. Different complex circumstance such as nonlinear dynamic loading, nonlinear structural modeling and any design constraints can be applied to this type of optimization procedure without any restrictions.

In genetic algorithm, a primary population is randomly generated and after that the fitness function will be checked. The good population will remain with a cross over rate and others are killed. After that a new population will be generated from the good ones. For prevention of stick answer in a local space, the algorithm is doing a mutation. In this specific problem, the fitness function is defined to be sum of damping coefficients in all stories. Primary population is generated randomly between 0 and 4000 (kN.sec/m). Before checking the fitness function, the constraint (MIDR should be less than acceptable values) should be controlled. If primary population satisfies the constraint, the algorithm will go on and if not, the population should be changed. After checking ET curve for MIDR with ASCE41-06 limit line, if a feasible solution is obtained, then the genetic algorithm tries to find the optimal solution; if not, new population of organisms should be reproduced. Because of negligible changes in MIDR under primary population, the algorithm may never converge. The convergence criteria may be defined as the algorithm passes a specified number of iteration or converge in damping coefficient is occurred.

5. Comparison of the Results of Optimization

The performance objective for BSE-1 level is considered to be LS (MIDR < 2.5%) and for BSE-2 level it is considered to be CP (MIDR < 5%). In other word, the design objectives 'p' and 'k' are assigned as the rehabilitation objectives [30]. As it was explained previously, another performance objective (UD, MIDR < 2.5%) is also defined for BSE-1 level. Scale factors for matched ground motions and equivalent time for acceleration functions are shown in Table (2). It should be noted that rehabilitation term is used for the optimum placement of dampers in the rest of paper for brevity.

A course of action should be done to obtain final ET curves for a frame. First, results of ET analysis are calculated for three acceleration functions. Then the absolute maximum of the response through all times of the analysis should be calculated. Finally, moving average of the results for three acceleration functions is computed. Moving average procedure makes the results smoother and more logical [15]. It should be noted that moving average procedure can be applied to each acceleration function individually or to the average results of three acceleration functions. Figure (4) shows the procedure for generating an ET curve for an individual acceleration function for frame S3B1. This is done for MIDR of this frame but it is also applicable for any other EDPs.

5.1. Optimization of Damper Distribution Using ET Analysis

By drawing the performance curve similar to Figure (4) and comparing the results with the allowable limits of the code, the vulnerability of each structure can be evaluated [18]. ET curves are continuous curves; and therefore, it is possible to compare the results at any time (equal to any intensity measure) with the allowable limits. Based
on the ET curve, genetic algorithm is used to optimize the damping coefficients to satisfy acceptable performance stated in the code, which is equivalent to make the performance curve (ET curve) below the allowable limit of the code. For example, required damping coefficients for S3B3 and S7B1 frames are shown in Figure (5). This figure shows that the second story of S3B3 frame and the first story of S7B1 frame are the best places for adding viscous dampers, and if these stories are controlled, then structural performance objectives can be achieved. In other words, these stories have an important role in the behavior of the frames, and their drift ratios will be appropriately controlled if damping coefficient shown in Figure (5) is provided there.

The performance curves of the structures before and after the rehabilitation are compared in Figures (6) to (8). Performance curves of the structures after rehabilitation is quite close to the allowable limit of the code and this indicates the optimal use of structure capacity. For S3B1 and S3B1-SSD frames, LS performance objective is dominant, and for S7B3-SSD frame CP performance objective is prevalent. Of course, the performance of these frames before rehabilitation is near the acceptable criteria, and it can be said that even for small changes in the structural properties, the optimization procedure can be used without any problem.

![Figure 5](image1.png)
![Figure 6](image2.png)
![Figure 7](image3.png)
![Figure 8](image4.png)

**Figure 5.** Optimum damper distribution for (a) S3B3 b) S7B1.

**Figure 6.** ET curve for S3B1 frame before and after rehabilitation.

**Figure 7.** ET curve for S3B1-SSD frame before and after rehabilitation.

**Figure 8.** ET curve for S7B3-SSD frame before and after rehabilitation.
5.2. Comparison of the Results of ET and Time History Analysis

The structural behavior of the frames under spectrum matched records is investigated in this section. Results are also compared with the ET method estimation. Figures (9) to (13) show the average response of the structure to seven spectrum matched records as well as the average response of the structure to ET acceleration functions before and after the rehabilitation of the frames.

Figure (9) shows the average interstory drift ratios of S3B1 frame at different stories for BSE-1 level. Besides, the average results plus and minus standard deviation is shown for time-history analysis. Although general trend of MIDR at different story levels are similar for two types of analysis, results are not the same. For example, results of time history analysis show that there is no need to add dampers to the frame because performance objective is satisfied for this frame at BSE-1 level, but MIDR obtained from ET analysis is greater than the acceptable value at the first story. Of course, the maximum difference between the results of two methods is about 15%. Response of the frame after rehabilitation is improved, and both analyses confirm this improvement.

Figure (10) shows the results for the same frame at BSE-2 level. The results of two analyses are more consistent in this excitation level. Both analyses show that S3B1 frame satisfies performance objective at BSE-2 level with no damper. Figure (6) also showed this result for ET analysis previously. ET method recognition for the best and the worst stories are similar to the results of time history analysis.

Figures (11) and (12) show the results of S7B1 frame at different stories for BSE-1 and BSE-2 levels respectively. This frame is strong enough to satisfy the performance objectives at these levels.

![Figure 9](image9.png)  
Figure 9. Average interstory drift ratios of S3B1 frame under GM set of ground motions and ET acceleration functions at BSE-1 level a) before rehabilitation, b) after rehabilitation.

![Figure 10](image10.png)  
Figure 10. Average interstory drift ratios of S3B1 frame under GM set of ground motions and ET acceleration functions at BSE-2 level a) before rehabilitation, b) after rehabilitation.
Evaluation of Endurance Time Analysis in Optimization of Viscous Damper Placement for Seismic Design

Figure 11. Average interstory drift ratios of S7B1 frame under GM set of ground motions and ET acceleration functions at BSE-1 level a) before rehabilitation, b) after rehabilitation.

Figure 12. Average interstory drift ratios of S7B1 frame under GM set of ground motions and ET acceleration functions at BSE-2 level a) before rehabilitation, b) after rehabilitation (considering UD performance objective).

Figure 13. Median and average interstory drift ratios of S7B3-SSD frame under GM set of ground motions and ET acceleration functions respectively at BSE-2 level a) before rehabilitation b) after rehabilitation.
before rehabilitation. Therefore, as it was discussed before, UD performance objective is used for this frame, and its MIDR is limited to 2.5% at BSE-2 level by adding viscous dampers. Unlike S3B1 frame, results of time-history analysis of S7B1 frame at both excitation levels are greater than ET analysis results. Figure (12) shows that although S7B1 frame satisfies UD performance objective after rehabilitation based on ET analysis results, this frame still have some problems in the lower stories based on the results of time history analysis. Again, differences between the results of two analyses are not too much and are less than 10%.

Figure (13) shows the results of S7B3-SSD frame at different stories for BSE-2 level. For this frame, time-history analyses did not converge for three records before rehabilitation (RRS, PET and LCN records). Therefore, median values of MIDR are shown for time-history analysis. Even in this condition, results of two analyses are compatible. After rehabilitation, time-history analysis did not converge just for one record (RRS record). Again, compatibility of the results is satisfactory after rehabilitation. Although this frame satisfies CP performance objective after rehabilitation, its problems at lower stories are not resolved. In other words, to reach a uniform distribution of interstory drift at different stories, it is necessary to optimize the design of the frame. Figure (13) shows that even for more complicated nonlinear models, compatibility of ET method and time-history analysis is satisfactory.

Figure (14) compares ET curves of S7B3 and S7B3-SSD frames after rehabilitation. The structural behavior of two frames is similar at low intensities, but after 10 seconds ET curves diverge from each other. Stiffness degradation and strength deterioration considered in S7B3-SSD frame increase its MIDR at higher intensities drastically, and ET analysis does not have converge after 20 seconds. This is comparable with the results of time-history analysis for this frame because for one record, this analysis did not converge at BSE-2 level and it is obvious that time-history analysis will not converge for more records at higher intensities. In other words, even by adding dampers and satisfying performance objectives, it cannot be guaranteed that the performance of the frame is ideal in higher intensities. Fortunately, all of these conclusions can be made just by three nonlinear dynamic analyses which show the advantage of ET method. Figure (15) compares ET and time-history analysis results for S7B3 and S7B3-SSD frames after rehabilitation. Like ET analysis, MIDR of S7B3-SSD frame is greater than S7B3 frame. Although distribution of interstory drift ratios at different stories is different for these frames, ET method can predict this distribution for both frames successfully. In other words, estimation of ET analysis for different structural models is convincing.

Figure (16) shows moment-rotation curves for one of the hinges of S7B3-SSD frame obtained from ET method and nonlinear dynamic analysis after rehabilitation. Figure (16a) belongs to 'ETA40g02' which shows rotation versus moment of the hinge before and after 15 sec in different colors. This separation is done because ET curve of this frame, Figure (14), shows that after 15 seconds interstory
drift ratio drastically increases, showing the effects of stiffness degradation and strength deterioration. This can also be seen in Figure (16a). Figure (16b) is for ‘H-E’ record at BSE-2 level that is equivalent to 15 seconds in ET analysis. Although ET curve shows fewer cycles than nonlinear dynamic analysis, maximum values for both methods are close to each other.

6. Conclusions

In ET method, structural performance can be assessed by a single numerical simulation at different seismic hazard levels. This will significantly reduce the computational effort as compared to IDA. In this paper, this advantage of ET method is used to find the optimum placement of dampers in steel frames. Genetic algorithm is applied to solve this optimization problem. Four steel moment-resisting frames are modeled using nonlinear beam-column elements without any deterioration in their characteristics and other four frames are modeled using concentrated plasticity, which considers stiffness degradation and strength deterioration of steel connections. Results of ET analysis are compared with the results of time-history analysis using seven near-field ground motions. Because ET acceleration functions used in this study are generated based on ASCE7-05 design spectrum, earthquake records are spectrally matched to this spectrum. Optimization is done to reach the target of three performance objectives.

Results of two analyses are compatible for all frames and distribution of interstory drift ratios at different stories is similar. In some frames, estimation of ET method for MIDR is greater than time-history analysis results; and therefore, rehabilitated frame based on ET method satisfies performance objective based on both analyses results for these cases. However, for some frames, it is vice versa; and therefore, a margin of safety should be used for optimization procedure to assure the optimal frame based on ET analysis results. The maximum difference between the results of two methods is about 15% in this study. Results of ET analysis are consistent with the results of time-history analysis at BSE-1 and BSE-2 levels, and results are also reliable even for frames that experience collapse for some of records. Of course, validity of ET method at higher intensities which propel the structures to collapse level needs more investigation.

The results of this investigation show that ET method can be used as an effective procedure to optimize damper placement in structures. Reduction of analysis time and availability of structural performance at different seismic hazard levels are the main advantages of this method. Reliability of ET method in solving this optimization problem can be achieved if a proper margin of safety is used for satisfying performance objectives.
Acknowledgement

The authors would like to thank University of Isfahan for their support on this research.

References


